

Définition :

A est un nombre positif ou nul, il existe un unique nombre a positif ou nul tel que : $a^2 = A$.
 a est appelé racine carrée de A et se note \sqrt{A} .

On peut alors écrire, pour tout A réel positif ou nul, $(\sqrt{A})^2 = A$.

Règles de calcul

Opérations	Conditions	Résultats
Produit de deux racines	$a \geq 0$ $b \geq 0$	$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$
Quotient de deux racines	$a \geq 0$ $b > 0$	$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$
Puissance d'une racine	n entier naturel $a \geq 0$	$(\sqrt{a})^n = \sqrt{a^n}$

Propriété :

Si $a \geq 0$, $\sqrt{a^2} = a$

Si $a \leq 0$, $\sqrt{a^2} = -a$

Pour simplifier une expression comportant un radical, il faut décomposer le nombre sous le radical en un produit comportant des carrés parfaits.

Liste des carrés parfaits à connaître :

$2^2 = 4$; $3^2 = 9$; $4^2 = 16$; $5^2 = 25$; $6^2 = 36$; $7^2 = 49$; $8^2 = 64$; $9^2 = 81$;
 $10^2 = 100$; $11^2 = 121$; $12^2 = 144$; $13^2 = 169$; $14^2 = 196$; $15^2 = 225$

Exemples: $\sqrt{63} = \sqrt{9 \times 7} = \sqrt{9} \times \sqrt{7} = \sqrt{3^2} \times \sqrt{7} = 3\sqrt{7}$

$\sqrt{32} = \sqrt{16 \times 2} = \sqrt{16} \times \sqrt{2} = \sqrt{4^2} \times \sqrt{2} = 4\sqrt{2}$

Exercices:

Exercice 1: Simplifier les expressions

A = $\sqrt{16}$ B = $\sqrt{12}$ C = $\sqrt{36}$ D = $\sqrt{20}$ E = $\sqrt{48}$ F = $\sqrt{50}$ G = $\sqrt{49}$

H = $\sqrt{72}$ I = $\sqrt{27}$ J = $\sqrt{125}$ K = $\sqrt{80}$ L = $\sqrt{48}$ M = $\frac{\sqrt{81}}{\sqrt{25}}$ N = $\frac{\sqrt{24}}{\sqrt{150}}$

Exercice 2: Simplifier

A = $\sqrt{24} \times \sqrt{30}$

B = $\sqrt{35} \times \sqrt{14}$

C = $\sqrt{21} \times \sqrt{14} \times \sqrt{50}$

D = $2\sqrt{2} + 4\sqrt{18}$

E = $3\sqrt{6} + 7\sqrt{24} - 5\sqrt{54}$

F = $2\sqrt{27} - 5\sqrt{3} + 4\sqrt{8}$

G = $7\sqrt{12} + 9\sqrt{5} + 8\sqrt{20} - 6\sqrt{27}$

H = $12\sqrt{8} - 9\sqrt{50} + 7\sqrt{200} + \sqrt{128}$

I = $6\sqrt{48} - 9\sqrt{75} + 7\sqrt{45} - 2\sqrt{27}$

J = $2\sqrt{8} - 5\sqrt{32} + 3\sqrt{800} + \sqrt{162}$

Correction

Exercice 1: Simplifier les expressions

$$A = \sqrt{16} = \sqrt{4^2} = 4$$

$$C = \sqrt{36} = \sqrt{6^2} = 6$$

$$E = \sqrt{48} = \sqrt{16 \times 3} = \sqrt{4^2} \times \sqrt{3} = 4\sqrt{3}$$

$$G = \sqrt{49} = \sqrt{7^2} = 7$$

$$I = \sqrt{27} = \sqrt{3 \times 3 \times 3} = \sqrt{3 \times 3^2} = 3\sqrt{3}$$

$$K = \sqrt{80} = \sqrt{2^2 \times 2^2 \times 5} = 4\sqrt{5}$$

$$M = \frac{\sqrt{81}}{\sqrt{25}} = \frac{\sqrt{9^2}}{\sqrt{5^2}} = \frac{9}{5}$$

$$B = \sqrt{12} = \sqrt{3 \times 2 \times 2} = \sqrt{3 \times 2^2} = 2\sqrt{3}$$

$$D = \sqrt{20} = \sqrt{5 \times 2 \times 2} = \sqrt{5 \times 2^2} = 2\sqrt{5}$$

$$F = \sqrt{50} = \sqrt{5 \times 2 \times 5} = \sqrt{2 \times 5^2} = 5\sqrt{2}$$

$$H = \sqrt{72} = \sqrt{3^2 \times 2^2 \times 2} = 3 \times 2\sqrt{2} = 6\sqrt{2}$$

$$J = \sqrt{125} = \sqrt{5 \times 5 \times 5} = \sqrt{5 \times 5^2} = 5\sqrt{5}$$

$$L = \sqrt{48} = \sqrt{4^2 \times 3} = 4\sqrt{3}$$

$$N = \frac{\sqrt{24}}{\sqrt{150}} = \frac{\sqrt{2^2 \times 2 \times 3}}{\sqrt{5^2 \times 2 \times 3}} = \frac{2}{5}$$

Exercice 2: Simplifier

$$A = \sqrt{24} \times \sqrt{30} = \sqrt{2^2 \times 6 \times 6 \times 5} = \sqrt{2^2 \times 6^2 \times 5} = 2 \times 6\sqrt{5} = 12\sqrt{5}$$

$$B = \sqrt{35} \times \sqrt{14} = \sqrt{7 \times 5 \times 7 \times 2} = \sqrt{7^2 \times 2 \times 5} = 7\sqrt{10}$$

$$C = \sqrt{21} \times \sqrt{14} \times \sqrt{50} = \sqrt{3 \times 7 \times 7 \times 2 \times 5^2 \times 2} = \sqrt{7^2 \times 5^2 \times 2^2 \times 3} = 7 \times 5 \times 2\sqrt{3} = 70\sqrt{3}$$

$$D = 2\sqrt{2} + 4\sqrt{18} = 2\sqrt{2} + 4\sqrt{3^2 \times 2} = 2\sqrt{2} + 4 \times 2\sqrt{2} = 2\sqrt{2} + 8\sqrt{2} = 10\sqrt{2}$$

$$E = 3\sqrt{6} + 7\sqrt{24} - 5\sqrt{54} = 3\sqrt{6} + 7\sqrt{2^2 \times 6} - 5\sqrt{6 \times 3^2} = 3\sqrt{6} + 14\sqrt{6} - 15\sqrt{6} = 2\sqrt{6}$$

$$F = 2\sqrt{27} - 5\sqrt{3} + 4\sqrt{8} = 2\sqrt{3 \times 3^2} - 5\sqrt{3} + 4\sqrt{2 \times 2^2} = 2 \times 3\sqrt{3} - 5\sqrt{3} + 4 \times 2\sqrt{2} = 6\sqrt{3} - 5\sqrt{3} + 8\sqrt{2} \\ = \sqrt{3} + 8\sqrt{2}$$

$$G = 7\sqrt{12} + 9\sqrt{5} + 8\sqrt{20} - 6\sqrt{27} = 7\sqrt{2^2 \times 3} + 9\sqrt{5} + 8\sqrt{2^2 \times 5} - 6\sqrt{3^2 \times 3} \\ = 14\sqrt{3} + 9\sqrt{5} + 16\sqrt{5} - 18\sqrt{3} \\ = -4\sqrt{3} + 25\sqrt{5}$$

$$H = 12\sqrt{8} - 9\sqrt{50} + 7\sqrt{200} + \sqrt{128} = 12\sqrt{2^2 \times 2} - 9\sqrt{5^2 \times 2} + 7\sqrt{2 \times 10^2} + \sqrt{8^2 \times 2} \\ = 24\sqrt{2} - 45\sqrt{2} + 70\sqrt{2} + 8\sqrt{2} \\ = 57\sqrt{2}$$

$$I = 6\sqrt{48} - 9\sqrt{75} + 7\sqrt{45} - 2\sqrt{27} = 6\sqrt{4^2 \times 3} - 9\sqrt{5^2 \times 3} + 7\sqrt{3^2 \times 5} - 2\sqrt{3^2 \times 3} \\ = 24\sqrt{3} - 45\sqrt{3} + 21\sqrt{5} - 6\sqrt{3} \\ = -6\sqrt{3}$$

$$J = 2\sqrt{8} - 5\sqrt{32} + 3\sqrt{800} + \sqrt{162} = 2\sqrt{2^2 \times 2} - 5\sqrt{4^2 \times 2} + 3\sqrt{10^2 \times 2^2 \times 2} + \sqrt{2 \times 9^2} \\ = 4\sqrt{2} - 20\sqrt{2} + 60\sqrt{2} + 9\sqrt{2} \\ = 53\sqrt{2}$$